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**OR**To prove that, for any two elements  $a, b$  of a Boolean algebra

- (a)  $a + b = \text{least upper bound of } a \text{ and } b$   
     i.e.  $a + b = \text{lub } (a, b)$
- (b)  $a \cdot b = \text{greatest lower bound of } a, b$   
     i.e.  $a \cdot b = \text{glb } (a, b)$

Q.8. If  $A, B, C$  are three sets then prove that:

- (i)  $A - (B \cap C) = (A - B) \cup (A - C)$
- (ii)  $A - (B \cup C) = (A - B) \cap (A - C)$

**OR**If  $R$  is an equivalence relation in a set  $A$  then prove that  $R^{-1}$  is equivalence relation in the set  $A$ 

Q.9. If  $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$  show that the function is continuous at  $x = 0$  but not differentiable at  $x = 0$

**OR**

Apply Maclaurian's theorem to prove that

$$\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \dots$$

Q.10. (a) Evaluate  $\int \frac{\log(\log x)}{x} dx$      (b) Solve  $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

**OR**Prove that  $\int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2} \log 2$ 

Q.11. If  $x^x y^y z^z = c$  then show that  $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$

**OR**

If  $u = \sin^{-1} \left( \frac{x^2 + y^2}{x+y} \right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$

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**B.C.A. FIRST SEMESTER EXAMINATION  
NOVEMBER – DECEMBER 2016**
**PAPER – II  
MATHEMATICAL FOUNDATION**

Time: 3hrs

Max. Marks: 100

Min. Marks: 40

Note: All sections are compulsory. Marks are indicated against each section.

**SECTION – A  
Objective Type Questions**

Q.1 Choose the correct answer. (2 x 10 = 20)

(i) By De Morgan's law for any two elements  $a$  and  $b$  of Boolean algebra  $B$ , which is true?

- (a)  $(a + b)' = a' \cdot b'$      (b)  $(a \cdot b)' = a + b'$   
     (c)  $(a + b)' = a' \cdot b$      (d)  $(a \cdot b)' = a' + b'$

(ii) The simplified function for switching function  $F(x, y) = x + xy$  is:

- (a)  $y$      (b)  $xy$      (c)  $x$      (d)  $x + y$

(iii) If a finite set  $A$  has  $n$  elements, then the power set of  $A$  has:  
     (a)  $2^{n+1}$  elements     (b)  $2^{n-1}$  elements  
     (c)  $2^n$  elements     (d)  $2^{2^n}$  elements(iv) If  $A = \{1, 2\}$   $B = \{2, 3\}$  then  $A \times B$  is equal to:

- (a)  $\{(1, 2), (1, 3), (2, 2), (3, 3)\}$   
     (b)  $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$   
     (c)  $\{(1, 2), (2, 3), (3, 1), (3, 2)\}$      (d) None of these

(v)  $\sin x =$ 

- (a)  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$      (b)  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$   
     (c)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$      (d)  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

P.T.O.

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- (vi)  $D^n x^n$  is equal to:  
 (a)  $(n - 1)!$     (b)  $n!$     (c)  $(n + 1)!$     (d)  $x^{n+1}$

- (vii)  $\int \sec x \, dx =$   
 (a)  $\log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$     (b)  $\log \cot\left(\frac{\pi}{4} + \frac{x}{2}\right)$   
 (c)  $\log(\sec x + \cot x)$     (d) None of these

- (viii)  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx =$   
 (a)  $\frac{\pi}{2}$     (b)  $\frac{\pi}{6}$     (c)  $\frac{\pi}{4}$     (d)  $\pi$

- (ix) If  $u = ax^2 + by^2 + 2hxy$  then value of  $\frac{\partial^2 u}{\partial x \partial y} =$   
 (a)  $2h$     (b)  $2$     (c)  $2x$     (d)  $2y$

- (x) If  $u = f\left(\frac{y}{x}\right)$  then the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is:  
 (a)  $0$     (b)  $-u$     (c)  $u$     (d)  $2u$

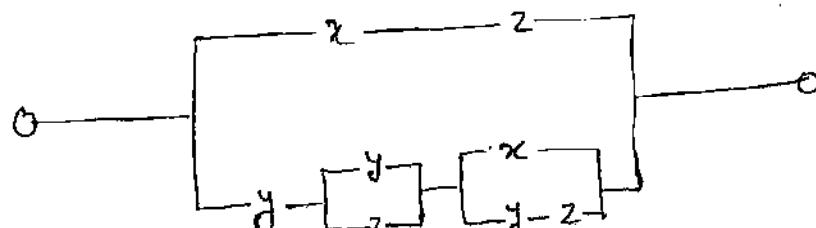
### SECTION – B Short Answer Type Questions

 $(6 \times 5 = 30)$ 

- Q.2. To show that  $(a \cdot b)' = a' + b'$

**OR**

Find the Boolean function of the following circuit and design in simplified form:



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- Q.3. If A, B, C are three sets, prove that  $(A - B) - C = A - (B \cup C)$

**OR**

Show that the relation  $R = \{(a, b) : a, b \in I \text{ and } a - b \text{ is divisible by } 5\}$  is an Equivalence relation.

- Q.4. If  $f(x) = |x - 2|$  then test the function  $f(x)$  for continuity at  $x = 2$ .

**OR**

Verify Rolle's theorem for the function  $f(x) = x^3 - 6x^2 + 11x - 6$

- Q.5. Evaluate  $\int \tan^{-1} \sqrt{x} \, dx$

**OR**

$$\text{Evaluate } \int \frac{xe^x}{(x-1)^2} \, dx$$

- Q.6. If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$  prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

**OR**

If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

### SECTION – C Long Answer Type Questions

 $(10 \times 5 = 50)$ 

- Q.7. Prove that the order relation ' $\leq$ ' is a partial order relation in a Boolean Algebra.

P.T.O.