

ES-3706**M. A. / M. Sc. (Fourth Semester) Special Examination, 2020****MATHEMATICS***Paper : First***(Abstract Harmonic Analysis)***Maximum Marks : 35**Note: Attempt all questions. Each question carries equal marks. Symbols have thier usual meanings.***1.** Prove that :

$$(T_a f) \wedge (n) = e^{-ina} \hat{f}(n) \text{ for } (n \in Z) \text{ and}$$

$$f \in L'.$$

2. If $f \in L'$ and $g \in C^k$. Then prove that $D^m(f * g) = f * D^m g$ for any integer $m \geq 0$ not exceeding K .**3.** Explain existence and uniqueness of Haar integral.**4.** Show that every compact abelian group G contains an infinite compact metric subgroup.**5.** The fourier transform, considered as a map of $L'(G)$ into $C^\circ(\Gamma)$ is norm decreasing and therefore continuous

$$\| \hat{f} \|_{\infty} \leq \| f \|_1.$$