

## ES-4306

**M. A./M. Sc. (Final) Special Examination, 2020**

**(For Private Students)**

**MATHEMATICS**

***Paper : II (Optional)***

**(Abstract Harmonic Analysis)**

***Maximum Marks : 100***

*Note: Attempt all questions. Each question carries equal marks.*

1. (a) Describe Banach space of continuous function and  $L^P$  space ( $1 \leq P < \infty$ ).  
(b) Define Translate and characters.
2. (a) For any LCA group  $G$ , prove that  $L^1(G)$ , is commutative Banach Algebra if multiplication is defined by convolution.  
(b) Define locally compact Abelian group.
3. (a) If  $f \in L^1$  has a Fourier series that is dominatelly convergent almost everywhere, then

$$f(x) = \sum_{n \in \mathbb{Z}} f(n) e^{inx} \text{ a.e.}$$

- (b) State and prove uniqueness theorem for trigonometric polynomials.
4. (a) Let  $Y$  be any nontrivial continuous complex homomorphism of  $L^1$ , then there exists a unique  $n \in \mathbb{Z}$  such that  $Y \rightarrow Y_n$  where  $Y_n(f) = f(n)$  for  $f \in L^1$ .  
(b) Prove that every bounded linear functional on  $L$  is the difference to two bounded, non negative, positive homogeneous, additive functions.
5. (a) Describe Fourier Stieltjes transform of measure and their properties.  
(b) Prove that if  $G$  is discrete then  $\hat{G}$  is compact.