ES-3712

M. A. / M. Sc. (Fourth Semester) Special Examination, 2020

MATHEMATICS

Paper : VII : Gr.-B (Opt.-I)

(Integration Theory)

Maximum Marks : 35

Note: *Attempt all questions. Each question carries equal marks.*

1. Let f be an extended real valued function where domain is measurable then the following statements are equivalent prove :

(i) for each real $\alpha, \{x : f(x) > \alpha\}$ is measurable.

(ii) for each real $\alpha, \{x : f(x) \geq \alpha\}$ is measurable.

(iii) for each real $\alpha, \{x : f(x) < \alpha\}$ is measurable.

(iv) for each real $\alpha, \{x : f(x) \leq \alpha\}$ is measurable.

2. State and prove Hahn Decomposition theorem.

3. State and prove Lebesgue's decomposition theorem.

4. Let μ be a Bair measure on a locally compact space X and E be a σ bounded Baire set in X, then for $\in > 0$ prove that :

(i) there is a σ-compact open set 0 with $E \subset 0$ and $\mu(0 \sim E) < \in$.

(ii) $\mu E = \sup\{\mu k : K \subset E,\ K \text{ is a compact } C_{i\delta}\}$

5. State and prove Riesz-Markov theorem.