ES-4313

M. A./M. Sc. (Final) Special Examination, 2020

(For Private Students)

MATHEMATICS

Paper : XII (Optional)

(Spline Theory)

Maximum Marks : 100

Note: *Attempt all questions. Each question carries equal marks.*

1. (a) Prove that if $\phi_1, \phi_2, \ldots\ldots, \phi_n$ is linearly independent and $A := (a_{ij})$ is $m \times n$ matrix whose in rows are linearly independent the sequence $\psi_1, \psi_2, \ldots\ldots, \psi_n$, with $\psi_n = \sum a_{ij}\phi_j$, all i is also linearly independent.

(b) Explain the support of B-spline.

2. (a) Discuss different boundary conditions in terms of spline interpolation.

(b) If g is twice differentiable, then prove that the second derivative of the interpolation error $e := g - I_4 g$ is orthogonal to $\$_2$.

3. (a) State and prove Curry Schoenberg theorem.

(b) Explain Recurrence relation for B splines.

4. (a) If g is a polynomial of degree $\leq K$, then prove that $[\tau_i \ldots\ldots \tau_{i+k}]g$ is constant as a function of $\tau_i \ldots\ldots \tau_{i+k}$.

(b) Explain limitation of polynomial approximation with an example.

5. (a) Prove that the least square approximation $L_2 g$ to $g \in c[a, b]$ by elements of $\$_2$ satisfied $\| L_2 g \| \leq 3 \| g \|$.

(b) Show that broken line interpolation is nearly optimal.