

**ES-3717****M. A./M. Sc. (Fourth Semester) Special Examination, 2020****MATHEMATICS*****Paper : XII (Group B : Optional)*****(Wavelets Analysis)*****Maximum Marks : 35****Note: Attempt all questions. Each question carries equal marks. Symbols have their usual meanings.*

1. Assume that the In-Place Fast Haar Wavelet Transform of a sample  $\vec{s} = (s_0, s_1, s_2, s_3)$  produces the results (6, 2, 1, 2). Apply the inverse transform to reconstruct the sample  $\vec{s}$ .
2. Assume that for a sample with eight entries  $\vec{s} = (s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7)$ , the In-Place Haar Wavelet Transform is (5, 1, 0, -2, -3, 1, -1, 0). Find the reconstruction of  $s_5$ .
3. State and prove Reverse Triangle Identity.
4. Show that the functions  $\omega_k : \mathbb{R} \rightarrow \mathbb{C}$  defined by  $\omega_{k(t)} := e^{\frac{ikt\pi}{T}}$  for each  $k \in \mathbb{Z}$  are orthonormal in  $C_{1,2T}^0(\mathbb{R}, \mathbb{C})$  with respect to the inner product  $\langle , \rangle$ .
5. Show that for each integrable weight function  $\omega : \mathbb{R} \rightarrow \mathbb{C}$  such that  $\lim_{|t| \rightarrow \infty} \omega(t) = 0$ , the set  $\{\omega_c : c > 0\}$  defined by  $\omega_c(t) := \frac{1}{c} \cdot \omega\left(\frac{t}{c}\right)$  constitutes an approximate identify.