A-6563

M. A. / M. Sc. (Final) Examination, April 2016

MATHEMATICS

Paper : XIII (Optional)

(Theory of Linear Operators)

Time Allowed : Three hours

Maximum Marks : 100

Note : *Attempt five questions in all, selecting* ***one*** *question from each unit. All questions carry equal marks. Symbols have their usual meaning.*

Unit-I

a) Define Banach Algebra. Show that the linear space P_{n+1} of all polynomials, of degree less than or equal to n, with complex coefficients is a Banach Algebra.

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(b) Define spectrum of an element of a complex Banach Algebra. Let $M_{2\times2}$ be the real Banach Algebra with identity. Find spectrum of A where

$$A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

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2. (a) Let $T\in B(X,X)$, where X is a Banach space. If $\|T\|<1$, then prove that $(I-T)^{-1}$ exists as a bounded linear operator on the whole space X and

$$(I-T)^{-1}=\sum_{n=0}^{\infty}T^{n}=I+T+T^{2}+\ldots\ldots\ldots.$$

(b) Let X be a Banach algebra and $x\in X$. Then show that spectral radius of $x=r(x)\leq\|x\|$.

Unit-II

3. (a) Define compact linear operator. Let X and Y b normed spaces then prove that every compact linea operator T from X to Y is bounded.

(b) Let $T:X\to X$ be a compact linear operator on

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Banach space X. Then prove that every spectral value $\lambda \neq 0$ of T_x is an eigen value of T.

4. Show that the set of eigen values of a compact linear operator T from X to X is countable and the only possible point of accumulation is $\lambda = 0$.

Unit-III

5. (a) Define Fredholm alternative.

(b) Let $T: H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space H then show that all the eigen values of T (if they exist) are real.

6. (a) Prove that the spectrum $\sigma(T)$ of a bounded self-adjoint linear operator $T: H \to H$ on a complex Hilbert space H lies in $[m, M]$ on the real axis, where

$$m = \inf_{\|x\|=1} \langle T_x, x \rangle$$

$$M = \sup_{\|x\|=1} \langle T_x, x \rangle$$

(b) Let $T: l^2 \to l^2$ be defined by

$(\xi_1, \xi_2, \xi_3, \ldots\ldots) \to (0, 0, \xi_1, \xi_2 \ldots\ldots)$ then show that T is positive and find a square root of T.

Unit-IV

7. (a) Explain spectral family of a bounded self-adjoint linear operator.

(b) Explain spectral representation of a bounded self-adjoint linear operator.

8. Prove that a bounded linear operator $P: H \to H$ on a Hilbert space H is a projection if and only if P is self-adjoint and $P^2 = P$.

Unit-V

9. (a) Define Hilbert-adjoint operator and prove that Hilbert-adjoint operator of a linear operator is linear.

(b) If a linear operator T is defined on all of a complex Hilbert space H and satisfies $\langle Tx, y \rangle = \langle x, Ty \rangle$ for all $x, y \in H$, then prove that T is bounded.

10. (a) Define symmetric operator and momentum operator.

(b) Write a note on time-independent Schrödinger equation.