inted Pages – 5

A-6559

M. A. / M. Sc. (Final) Examination, April 2016

MATHEMATICS

Paper : Sixth (Optional)

(Fundamental of Applied Functional Analysis)

Time Allowed : Three hours

Maximum Marks : 100

Note : Attempt all questions, selecting one question from each unit. All questions carry equal marks.

Unit-I

(a) Show that every closed convex set in a Hilbert Space has a unique element of minimal norm.

(b) Define Cartisian & Tensor Product of Hilbert Space.

Or

(a) Define :

(i) Convex set

(ii) Projection

(iii) Hilbert space

(iv) Inner product space

(b) C is closed convex set in H. Prove that z is the unique element in C closest to x if and only if :

$$\text{Re}\,[x-z,\ z-y] \geq 0 \ \forall\ y \text{ in } C.$$

Unit-II

2. (a) Define support functional & discuss its properties.

(b) Let C be a closed convex set in H with the origin as an interior point. Then show that for any x_0 on the boundary of C. We can find a support plane through x_0.

Or

(a) Show that a continuous convex functional defined o

a Hilbert space achieves its minimum on every convex closed bounded set.

(b) Define :

(i) Internal point

(ii) Extremal point

(iii) Closed convex hull

(iv) Support mapping

Unit-III

(a) State & prove Minimax theorem.

(b) Define :

(i) Dual of a convex cone

(ii) Fundamental of Game theory

Or

(a) State and prove Lagrange's Duality theorem.

(b) Define Vector maximum problem & discuss its properties.

Unit-IV

(a) Define Bounded & Unbounded Operator.

(b) Show that the spectrum of any closed operator is closed and spectrum of bounded operator can not be empty.

Or

(a) Define Fredholm alternative with an example.

(b) Suppose $\lambda \neq 0$ and the range $(\lambda I - T)$ is all of H.

Then λ is in the resolvent set of T.

Unit-V

5. (a) Define :

(i) Volterra operator

(ii) Hilbert Schmidt norm

(iii) Nuclear operator

(iv) Polar decomposition of an operator

(b) If A is a linear bounded operator mapping H_1 into H_2 and $T = \sqrt{A^* A}$ is compact and $\sum_{i=1}^{\infty} \lambda_0 < \infty$ where $\{\lambda_c\}$ are the eigen value of T. Then prove A is nuclear.

[5]

Or

(a) State & prove Tricome theorem.

(b) Define :

(i) Define Volterra Operator & discuss its compactness.

(ii) Define Hilbert-Schmidt Operator.