MTC-404

M. Sc. (Fourth Semester) Examination, 2020

(CBCS Course)

MATHEMATICS

(Spline Theory)

Maximum Marks : 60

Note: *Attempt all questions. All questions carry equal marks.*

1. (a) If $\tau_1 ....\tau_n$ are distinct points and $g(\tau_1)_1 ....g(\tau_n)$ are given data then prove that there exists exactly one polynomial $p \in \mathbb{P}_n$ for which $p(\tau_1) = g(\tau_i)$, $i = 1,.....n$. This polynomial can be written in language form

$$P = \sum_{i=1}^{n} g(\tau_i) li$$

with $li(x) := \prod_{j \neq i} (X - \tau_j) / (\tau_i - \tau_j)_1$ all i.

(b) If $g \in C^k$, i.e. g has K continuous derivatives, then prove that there exists a point ξ in the smallest interval containing $\tau_i, ...\tau_{i+k}$ so that

$$[\tau_i, ...\tau_{i+k}] g = g^{(k)}(\xi)/k!.$$

2. (a) Show that the Π_2 approximation $L_2 g$ to $g \in C[a,b]$, i.e. to a continuous function g on $[a,b]$, by elements of ϕ satisfies.

$\| L_2 g \| \leq 3 \| g \|$

Hence, since L_2 is additive and $L_2 f = f$ for all $f \in \phi_2$ we have $\| L_2 g \| \leq 4\ d_{is}(g, \$_2)$.

(b) Verify numerically that $\| g - I_2 g \| = \theta(n^{-2})$ in case $g(x) = \sin\sqrt{x}$ and $\tau_i = (i-1)/(n-1)^4$ $i = 1, 2, ...n$.

3. (a) Prove that an arbitrary cubic spline P can be written uniquely as :

$$P(x) = P(0)\phi_1(x) + P(h)\phi_2(x) + P'(0)\phi_3(x)$$
$$+ P'(h)\phi_4(x)$$

for given $h \neq 0$, with

$\phi_1(x) := 1 + y^2(2y-3)$

$\phi_2(x) := \phi_1(h-x) = 1 - \phi_1(x)$

$\phi_3(x) := y(h-x)^2/h,\ \phi_4(x); = -\phi_3(h-x)$

$y := X/h$ Hence

$P(x) = P(0) + (P(h) - P(0))y^2(3-2y) +$

$(P'(0)(1-y) - p'(h)y)y(h-x)$

(b) State and prove Best approximation property of cubic spline.

4. (a) Determine the existence and uniqueness of the parabolic spline $I_3 g$ for the given function g which agrees with $I_3 g$ at $\{\tau_i\}_{i=1}^{n}$.

(b) Write the polynomial representation for $f \in \mathbb{P}_{R_1}\ \xi$ and show that $\mathbb{P}_{R_1}\ \xi$ is a linear space of dimension Kl.

5. (a) Show that truncated power function forms the basis for $\mathbb{P}_{R_1}\ \xi$ and $\mathbb{P}_{R_1}\ \xi, \upsilon$ spaces.

(b) Show that $\{\beta_i\}$ (B - spline) provides partition of unity.